CSIR NET/JRF

Mathematical Science

16 Feb. 2022

PART-B

(Mathematical Sciences)

(21.) Consider the two statements given below:

I. There exists a matrix $N \in \mathbb{M}_4(\mathbb{R})$ such that $\{(1,1,1,-1),(1,-1,1,1)\}$ is a basis of $\text{Row}(N)$ and $(1,2,1,4) \in \text{Null}(N)$.

II. There exists a matrix $M \in \mathbb{M}_4(\mathbb{R})$ such that $\{(1,1,1,0)^T,(1,0,1,1)^T\}$ is a basis of $\text{Col}(M)$ and $(1,1,1,1)^T, (1,0,1,0)^T \in \text{Null}(M)$.

Which of the following statements is true?

(a.) Statement I is TRUE and Statement II is FALSE

(b.) Both Statement I and Statement II are FALSE

(c.) Both Statement I and Statement II are TRUE

(d.) Statement I is TRUE and Statement II is FALSE

(22.) $\lim\limits_{n\to\infty} \frac{1}{n}\left(1+\sqrt{2}+\sqrt[3]{3}+\ldots+\sqrt[n]{n}\right)$

(a.) Is equal to 0

(b.) Is equal to 1

(c.) Is equal to 2

(d.) Does not exist

(23.) Let $M = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & 3 \end{bmatrix}$. Given that 1 is an eigenvalue of M, which of the following statements is true?

(a.) -2 is an eigenvalue of M

(b.) 3 is an eigenvalue of M

(c.) The eigen space of each eigen value has dimension 1

(d.) M is diagonalizable

(24.) Let $S=\{1,2,\ldots,100\}$ and let $A=\{1,2,\ldots,10\}$ and $B=\{41,42,\ldots,50\}$. What is the total number of subsets of S, which have non-empty intersection with both A and B?

(a.) $\frac{2^{100}}{2^{20}}$

(b.) $\dfrac{100!}{10!10!}$

(c.) $2^{80}\left(2^{10}-1\right)^2$

(d.) $2^{100}-2\left(2^{10}\right)$

(25.) Let A be a 4×4 matrix such that $-1,1,1,-2$ are its eigenvalues. If $B=A^4-5A^2+5I$, then trace$(A+B)$ equals

(a.) 0

(b.) -12

(c.) 3

(d.) 9

(26.) Let $n>1$ be a fixed natural number. Which of the following is an inner product on the vector space of $n\times n$ real symmetric matrices?

(a.) $\langle A,B\rangle=(\text{trace}(A))(\text{trace}(B))$

(b.) $\langle A,B\rangle=\text{trace}(AB)$

(c.) $\langle A,B\rangle=\text{determinant}(AB)$

(d.) $\langle A,B\rangle=\text{trace}(A)+\text{trace}(B)$

(27.) Consider the sequence $\{a_n\}_{n>1}$, where $a_n=3+5\left(-\frac{1}{2}\right)^n+(-1)^n\left(\frac{1}{4}+(-1)^n\frac{2}{n}\right)$. Then the interval $\left(\liminf_{n\to\infty} a_n, \limsup_{n\to\infty} a_n\right)$ is given by

(a.) $(-2,8)$

(b.) $\left(\frac{11}{4},\frac{13}{4}\right)$

(c.) $(3,5)$

(d.) $\left(\frac{1}{4},\frac{7}{4}\right)$

(28.) Let $f,g:\mathbb{R}\mapsto\mathbb{R}$ be given by and $f(x)=x^2$ and $g(x)=\sin x$. Which of the following functions is uniformly continuous on $\mathbb{R}$?

(a.) $h(x)=g(f(x))$

(b.) $h(x)=g(x)f(x)$

(c.) $h(x)=f(g(x))$

(d.) $h(x)=f(x)+g(x)$

(29.) Let $S_1=\frac{1}{3}-\frac{1}{2}\times\frac{1}{3^2}+\frac{1}{3}\times\frac{1}{3^3}-\frac{1}{4}\times\frac{1}{3^4}+\dots$

and $S_2 = \frac{1}{4} + \frac{1}{2} \times \frac{1}{4^2} + \frac{1}{3} \times \frac{1}{4^3} + \frac{1}{4} \times \frac{1}{4^4} + \ldots$.

Which of the following identities is true?

(a.) $3S_1 = 4S_2$

(b.) $4S_1 = 3S_2$

(c.) $S_1 + S_2 = 0$

(d.) $S_1 = S_2$

(30.) Which of the following sets are countable?

(a.) The set of all polynomials with rational coefficients.

(b.) The set of all polynomials with real coefficients having rational roots.

(c.) The set of all 2×2 real matrices with rational eigenvalues

(d.) The set of all real matrices whose row echelon form has rational entries

(31.) Let A and B be $n \times n$ matrices. Suppose the sum of the elements in any row of A is 2 and the sum of the elements in any column of B is 2. Which of the following matrices is necessarily singular?

(a.) $I - \frac{1}{2} BA^T$

(b.) $I - \frac{1}{2} AB$

(c.) $I - \frac{1}{4} AB$

(d.) $I - \frac{1}{4} BA^T$

(32.) Let $V = \{A \in M_{3\times3}(\mathbb{R}) : A^t + A \in \mathbb{R} \cdot I\}$, where I is the identity matrix. Consider the quadratic form defined as $q(A) = \text{Trace}(A)^2 - \text{Trace}(A^2)$. What is the signature of this quadratic form?

(a.) $(++++)$

(b.) $(+0\,0\,0)$

(c.) $(+---)$

(d.) $(---0)$

(33.) Let γ be the positively oriented circle $\{z \in \mathbb{C} : |z| = 3/2\}$. Suppose that $\int_\gamma \frac{e^{i\pi z}}{(z-1)(z-2i)^2} dz = 2\pi i C$. Then $|C|$ equals

(a.) 2

(b.) 5

(c.) $1/2$

(d.) $1/5$

(34.) Let $\mathbb{D} \subset \mathbb{C}$ be the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{O}(\mathbb{D})$ be the space of all holomorphic functions on $\mathbb{D}$. Consider the sets $A = \left\{ f \in \mathcal{O}(\mathbb{D}) : f\left(\frac{1}{n}\right) = \begin{cases} e^{-n} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} ; \text{for } n \geq 2 \right\}$.

$B = \left\{ f \in \mathcal{O}(\mathbb{D}) : f\left(\frac{1}{n}\right) = (n-2)/(n-1), n \geq 2 \right\}$.

Which of the following statement is true?

(a.) Both A and B are non-empty

(b.) A is empty and B has exactly one element

(c.) A has exactly one element and B is empty

(d.) Both A, B are empty

(35.) How many generators does a cyclic group of order 36 have?

(a.) 6

(b.) 12

(c.) 18

(d.) 24

(36.) Which of following statements is necessarily true for a commutative ring R with unity?

(a.) R may have no maximal ideals

(b.) R can have exactly two maximal ideals

(c.) R can have one or more maximal ideals but no prime ideals

(d.) R has at least two prime ideals

(37.) Let $f(z)$ be a non-constant entire function and $z = x + iy$. Let $u(x, y), v(x, y)$ denote its real and imaginary parts respectively. Which of the following statements is FALSE?

(a.) $u_x = v_y$ and $u_y = -v_x$

(b.) $u_y = v_x$ and $u_x = -v_y$

(c.) $|f'(x+iy)|^2 = u_x(x,y)^2 + v_x(x,y)^2$

(d.) $|f'(x+iy)|^2 = u_y(x,y)^2 + v_y(x,y)^2$

(38.) Let $S = \{n : 1 \leq n \leq 999 ; 3 \mid n \text{ or } 37 \mid n\}$. How many integers are there in the set $S^c = \{n : 1 \leq n \leq 999 ;\ n \notin S\}$?

(a.) 639

(b.) 648

(c.) 666

(d.) 990

(39.) Let f be a rational function of a complex variable z given by $f(z)=\frac{z^3+2z-4}{z}$. The radius of convergence of the Taylor series of f at $z=1$ is

(a.) 0

(b.) 1

(c.) 2

(d.) ∞

(40.) Let (X,d) be a metric space and let $f:X\to X$ be a function such that $d(f(x),f(y))\le d(x,y)$ for every $x,y\in X$. Which of the following statements is necessarily true?

(a.) f is continuous

(b.) f is injective

(c.) f is surjective

(d.) f is injective if and only if f is surjective

(41.) If $y(x)$ is a solution of the equation $4xy''+2y'+y=0$ satisfying $y(0)=1$. Then $y''(0)$ is equal to

(a.) $1/24$

(b.) $1/12$

(c.) $1/6$

(d.) $1/2$

(42.) A body moves freely in a uniform gravitational field. Which of the following statements is true?

(a.) Stable equilibrium of the body is possible

(b.) Stable equilibrium of the body is not possible

(c.) Stable equilibrium of the body depends on the strength of the field

(d.) Equilibrium is metastable

(43.) Which of the following is an extremal of the functional $J(y)=\int_{-1}^{1}\left(y'^2-2xy\right)dx$ that satisfies the boundary conditions $y(-1)=-1$ and $y(1)=1$?

(a.) $-\frac{x^3}{5}+\frac{6x}{5}$

(b.) $-\frac{x^5}{8}+\frac{9x}{8}$

(c.) $-\frac{x^3}{6}+\frac{7x}{6}$

(d.) $-\frac{x^3}{7}+\frac{8x}{7}$

(44.) Let $a,b,c \in \mathbb{R}$ be such that the quadrature rule $\int_{-1}^{1} f(x)\,dx = af(-1) + bf'(0) + cf'(1)$ is exact for all polynomials of degree less than or equal to 2. Then $a+b+c$ equal to

(a.) 4

(b.) 3

(c.) 2

(d.) 1

(45.) Consider the following two initial value ODEs

(A) $\dfrac{dx}{dt} = x^3,\ x(0)=1;$

(B) $\dfrac{dx}{dt} = x\sin x^2,\ x(0)=2$

Related to these ODEs, we make the following assertions.

(I) The solution to (A) blows up in finite time.

(II) The solution to (B) blows up in finite time.

Which of the following statements is true?

(a.) Both I and II are true

(b.) I is true but II is false

(c.) Both I and II are false

(d.) I is false but II is true

(46.) Which of the following partial differential equations is NOT PARABOLIC for all $x,y \in \mathbb{R}$?

(a.) $x^2 \dfrac{\partial^2 u}{\partial x \partial y} - 2xy \dfrac{\partial u}{\partial y} + y^2 = 0$

(b.) $x^2 \dfrac{\partial^2 u}{\partial x^2} - 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 0$

(c.) $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 0$

(d.) $x^2 \dfrac{\partial^2 u}{\partial x^2} - 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} + x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$

(47.) Let $u(x,y)$ solve the Cauchy problem $\dfrac{\partial u}{\partial y} - x\dfrac{\partial u}{\partial x} + u - 1 = 0$ where $-\infty < x < \infty,\ y \ge 0$ and $u(x,0) = \sin x$. Then $u(0,1)$ is equal to

(a.) $1 - \dfrac{1}{e}$

(b.) $1 + \dfrac{1}{e}$

(c.) $1 - \dfrac{1 - \sin e}{e}$

(d.) $1+\frac{1-\sin e}{e}$

(48.) Let the solution to the initial value problem

$y' = y - t^2 + 1, 0 \le t \le 2,\ y(0) = 0.5$

be computed using the Euler's method with step-length $h = 0.4$. If $y(0.8)$ and $w(0.8)$ denote the exact and approximate solutions at $t = 0.8$, then an error bound for Euler's method is given by

(a.) $0.2(0.5e^2 - 2)(e^{0.4} - 1)$

(b.) $0.1(e^{0.4} - 1)$

(c.) $0.2(0.5e^2 - 2)(e^{0.8} - 1)$

(d.) $0.1(e^{0.8} - 1)$

(49.) Suppose Poisson $X|\lambda \sim \text{Poisson}(\lambda)$ where $\lambda > 0$. Consider the exponential distribution with mean $1/4$ for the prior on λ. If the observed value of X is 0, then which among the following is the 95% Bayesian credible (confidence) interval for λ of smallest length?

(a.) $(0,c)$ where $c = 0.95$ b

(b.) $(0,c)$ where $c = \frac{\log(20)}{5}$

(c.) $(c, \exp(c))$ where $c = \frac{\log(20)}{5}$

(d.) $(0.2 - c, 0.2 + c)$ where $c = \frac{\log(20)}{5}$

(50.) A proportion p of a large population has particular disease. A random sample of k people is drawn from the population and their blood samples are combined. An accurate test for the disease applied to the combined blood sample shows a positive result, hence at least one of the k people has the disease. What is the probability that exactly one of the k people has the disease?

(a.) $\frac{kp(1-p)^{k-1}}{1-(1-p)^k}$

(b.) $\frac{(1-p)^k + kp(1-p)^{k-1}}{1-(1-p)^k}$

(c.) $\frac{p}{p+p^2+\ldots+p^k}$

(d.) $\frac{1}{k}$

(51.) Let $S = \{1,2,3,4,5\}$. Consider a Markov chain on the state space S with transition probability matrix.

$$\begin{pmatrix} 0 & 0.3 & 0.7 & 0 & 0 \\ 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 \\ 0.7 & 0 & 0 & 0 & 0.3 \\ 0.3 & 0.7 & 0 & 0 & 0 \end{pmatrix}$$

Then which of the following is always true?

(a.) State 1 has period 2

(b.) State 2 is recurrent

(c.) State 3 is transient

(d.) The chain admits at least two stationary distributions

(52.) Consider a BIBD (Balanced Incomplete Block Design) with v treatments in b blocks, each of which has k plots. Let r denote the number of blocks in which each treatment occurs. Let λ be the number of blocks in which each pair of treatment occurs. Which of the following statements is necessarily true?

(a.) $vb = rk$

(b.) $vr = bk$

(c.) $r(b-1) = \lambda(k-1)$

(d.) $r(v-1) = \lambda(b-1)$

(53.) Let $X_1, X_2, \ldots, X_{16}$ be a random sample from normal distribution with unknown mean μ and variance 4. Suppose $Z \sim N(0,1)$. For the most powerful test for testing $H_0 : \mu = 3$ vs $H_1 : \mu = 0$, which one of the following is the p-value where the observed sample mean is 2.5

(a.) $p(Z > 1)$

(b.) $p(Z > -1)$

(c.) $p(Z > 0.5)$

(d.) $p(Z > -0.5)$

(54.) Let $X_1, X_2, \ldots$ be i.i.d. random variables with uniform distribution on the interval $[0,1]$. Let $Y_{n,k}$ denote the k^{th} order statistic based on the sample $X_1, \ldots, X_n$ (e.g. $Y_{n,1} = \min\{X_1, \ldots, X_n\}$) . What is the probability that $Y_{21,7} = Y_{22,7}$?

(a.) $\frac{1}{3}$

(b.) $\frac{2}{3}$

(c.) $\frac{7}{11}$

(d.) $\frac{15}{22}$

(55.) Let X_1, X_2, X_3, X_4 be i.i.d. random variables having uniform distribution on $(0,\theta)$ where $\theta > 0$ is an unknown parameter. Define $X_{(4)} = \max\{X_1, X_2, X_3, X_4\}$. Consider the confidence intervals $I = \left[2X_{(4)}, 3X_{(4)}\right]$ and $J = \left[X_{(4)}, 1 + X_{(4)}\right]$ for θ. Which of the following is true?

(a.) The converge probabilities of I and J are both independent of θ

(b.) The converge probability of I is independent of θ but the converge probability of J is NOT independent of θ.

(c.) The converge probability of J is independent of θ but the converge probability of I is NOT independent of θ.

(d.) The converge probabilities of both I and J are NOT independent of θ.

(56.) Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with mean θ. Which of the following is NOT a sufficient statistic for θ?

(a.) $\dfrac{1}{X_1 + X_2 + \ldots + X_n}$

(b.) $X_1 + X_2 + \ldots + X_n$

(c.) $\dfrac{X_n}{X_1 + X_2 + \ldots + X_{n-1}}$

(d.) $(X_n, X_1 + X_2 + \ldots + X_{n-1})$

(57.) Consider a $M/M/1$ queueing system with traffic intensity $p < 1$. The probability of having n customers in the system at the steady state is given by

(a.) ρ^n

(b.) $\rho(1-\rho^n)$

(c.) $\rho^{n-1}(1-\rho)$

(d.) $\rho^n(1-\rho)$

(58.) Suppose $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ are bivariate measurements where $n > 2$. Assume that all the x_i are distinct and all the y_i are distinct too. Let r_p denote the ordinary (Pearson) correlation coefficient and r_s denote the (Spearman) rank correlation coefficient. Suppose $r_p = 1$. Which of the following is true?

(a.) $0.5 < r_s < 1$

(b.) $r_s = 0.5$

(c.) $r_s = 1$

(d.) $r_s = -1$

(59.) A newly developed algorithm for random number generation need to be tested. The first step is to check whether the sequence of numbers generated can be considered a random sample from the uniform distribution on the interval $(0,1)$. Which of the following is an appropriate non-parametric test?

(a.) Wilcoxon signed rank test

(b.) Sign test

(c.) Paired t test

(d.) Kolmogorov-Smirnov test

(60.) Suppose that Y has Exponential distribution with mean θ and that the conditional distribution of X given $Y = y$ is Normal with mean 0 and variance y, for all $y > 0$. Identity the characteristic function of X (defined as $\phi(t) = \mathbb{E}[e^{itX}]$) from the following.

(a.) $e^{-\frac{\theta}{2}t^2}$

(b.) $e^{-\frac{1}{2\theta}t^2}$

(c.) $\dfrac{1}{1+\frac{1}{2}\theta t^2}$

(d.) $\dfrac{\theta}{\theta+\frac{1}{2}\theta t^2}$

PART-C

(Mathematical Sciences)

(61.) Which of the following are inner products on $\mathbb{R}^2$?

(a.) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + 2x_1y_2 + 2x_2y_1 + x_2y_2$

(b.) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$

(c.) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$

(d.) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 - \frac{1}{2}x_1y_2 - \frac{1}{2}x_2y_1 + x_2y_2$

(62.) Let $A \subseteq \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Which of the following statements are true?

(a.) If A is closed then $f(A)$ is closed

(b.) If A is bounded then $f^{-1}(A)$ is bounded

(c.) If A is closed and bounded then $f(A)$ is closed and bounded

(d.) if A is bounded then $f(A)$ is bounded

(63.) Let $T : X \to Y$ be a bounded linear operator from a Banach space X to another Banach space Y. Which of the following conditions imply that T has a bounded inverse?

(a.) $\inf_{\|x\|=1} \|Tx\| = 0$

(b.) $\inf_{\|x\|=1} \|Tx\| = 0$ and $T(X)$ is dense in Y

(c.) $\inf_{\|x\|=1} \|Tx\| > 0$

(d.) $\inf_{\|x\|=1} \|Tx\| > 0$ and $T(X)$ is dense in Y

(64.) Let A be an $m \times n$ matrix such that the first r rows of A are linearly independent and the first s columns of A are linearly independent, where $r < m$ and $s < n$. Which of the following statements are true?

(a.) The rank of A is at least $\max\{r, s\}$

(b.) The submatrix formed by the first r rows and the first s columns of A has rank $\min\{r, s\}$

(c.) If $r < s$, then there exists a row among rows $r+1, \ldots, m$ which together with the first r rows form a linearly independent set.

(d.) If $s < r$, then there exists a column among columns $s+1, \ldots, n$ which together with the firsts s columns form a linearly dependent set

(65.) Let $f : [0,1] \to \mathbb{R}$ be a continuous function such that $\int_0^t f(x)\,dx = \int_t^1 f(x)dx$, for every $t \in [0,1]$. Then which of the following are necessarily true?

(a.) f is differentiable on $(0,1)$

(b.) f is monotonic on $[0,1]$

(c.) $\int_0^1 f(x)dx = 1$

(d.) $f(x) > 0$ for all rationals $x \in [0,1]$

(66.) Consider the system

$$2x + ky = 2 - k$$

$$kx + 2y = k$$

$$ky + kz = k - 1$$

in three unknowns and one real parameter k. For which of the following values of k is the system of linear equation consistent?

(a.) 1

(b.) 2

(c.) -1

(d.) -2

(67.) For non-negative integers $k \geq 1$ define $f_k(x) = \dfrac{x^k}{(1+x)^2}\ \forall x \geq 0$.

Which of the following statements are true?

(a.) For each k, f_k is a function of bounded variation on compact intervals (b.) For every k, $\int_0^\infty f_k(x)\,dx < \infty$

(c.) $\lim_{k\to\infty}\int_0^1 f_k(x)\,dx$ exists

(d.) The sequence of functions f_k converge uniformly on $[0,1]$ as $k\to\infty$

(68.) Let $\mathbb{R}^+$ denote set of all positive real numbers. Suppose that $f:\mathbb{R}^+ \mapsto \mathbb{R}$ is a differentiable function. Consider the function $g(x)=e^x f(x)$. Which of the following are true?

(a.) If $\lim_{x\to\infty} f(x)=0$ then $\lim_{x\to\infty} f'(x)=0$

(b.) If $\lim_{x\to\infty}\left(f(x)+f'(x)\right)=0$ then $\lim_{\substack{x\to\infty\\ y\to\infty}}\frac{g(x)-g(y)}{e^x-e^y}=0$

(c.) If $\lim_{x\to\infty} f'(x)=0$ then $\lim_{x\to\infty} f(x)=0$

(d.) If $\lim_{x\to\infty}\left(f(x)+f'(x)\right)=0$ then $\lim_{x\to\infty} f(x)=0$

(69.) Let A be an $m\times m$ matrix with real entries and let x be an $m\times 1$ vector of unknowns. Now consider the two statements given below:

I : There exists non-zero vector $b_1\in\mathbb{R}^m$ such that the linear system $Ax=b_1$ has NO solution.

II : There exist non-zero vectors $b_2, b_3\in\mathbb{R}^m$, with $b_2\neq cb_3$ for any $c\in\mathbb{R}$, such that the linear systems $Ax=b_2$ and $Ax=b_3$ have solutions.

Which of the following statements are true?

(a.) II is TRUE whenever A is singular

(b.) I is TRUE whenever A is singular

(c.) Both I and II can be TRUE simultaneously

(d.) If $m=2$, then at least one of I and II is FALSE

(70.) Let $f:\mathbb{R}^3\to\mathbb{R}^2$ be a C^1 function with $f(0,0,0)=(0,0)$. Let A denote the derivative of f at $(0,0,0)$. Let $g:\mathbb{R}^3\to\mathbb{R}$ be the function given by $g(x,y,z)=xy+yz+zx+x+y+z$. Let $h:\mathbb{R}^3\to\mathbb{R}^3$ be the function define by $h=(f,g)$. In which of the following cases, will the function h admit a differentiable inverse in some open neighbourhood of $(0,0,0)$?

(a.) $A=\begin{pmatrix}1&2&1\\0&0&0\end{pmatrix}$

(b.) $A=\begin{pmatrix}2&2&2\\6&5&2\end{pmatrix}$

(c.) $A=\begin{pmatrix}0&0&-1\\0&1&0\end{pmatrix}$

(d.) $A=\begin{pmatrix}4&2&4\\0&3&2\end{pmatrix}$

(71.) Let X be a topological space and E be a subset of X. Which of the following statements are correct?

(a.) E is connected implies $\bar{E}$ is connected

(b.) E is connected implies ∂E is connected

(c.) E is path connected implies $\bar{E}$ is path connected

(d.) E is compact implies $\bar{E}$ is compact

(72.) It is known that $X = X_0 \in M_2(\mathbb{Z})$ is a solution of $AX - XA = A$ for some $A \in \left\{ \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right\}$. Which of following values are NOT possible for the determinant of X_0 ?

(a.) $\det(X_0) = 0$

(b.) $\det(X_0) = 2$

(c.) $\det(X_0) = 6$

(d.) $\det(X_0) = 10$

(73.) Let $M \in \mathbb{M}_n(\mathbb{R})$ such that $M \neq 0$ but $M^2 = 0$. Which of the following statements are true?

(a.) If n is even then $\dim(\mathrm{Col}(M)) > \dim(\mathrm{Null}(M))$

(b.) If n is even then $\dim(\mathrm{Col}(M)) \leq \dim(\mathrm{Null}(M))$

(c.) If n is odd then $\dim(\mathrm{Col}(M)) < \dim(\mathrm{Null}(M))$

(d.) If n is odd then $\dim(\mathrm{Col}(M)) > \dim(\mathrm{Null}(M))$

(74.) In which of the following cases does there exist a continuous and onto function $f : X \to Y$?

(a.) $X = (0,1), Y = (0,1]$

(b.) $X = [0,1], Y = (0,1]$

(c.) $X = (0,1), Y = \mathbb{R}$

(d.) $X = (0,2), Y = \{0,1\}$

(75.) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a bounded function such that for each $t \in \mathbb{R}$, the function g_t and h_t given by $g_t(y) = f(t,y)$ and $h_t(x) = f(x,t)$ are non decreasing functions. Which of the following statements are necessarily true?

(a.) $k(x) = f(x,x)$ is a non-deceasing function

(b.) Number of discontinuities of f is at most countably infinite

(c.) $\lim_{(x,y)\to(+\infty,+\infty)} f(x,y)$ exists

(d.) $\lim_{(x,y)\to(+\infty,-\infty)} f(x,y)$ exists

(76.) Let Y be a nonempty bounded, open subset of $\mathbb{R}^n$ and let $\bar{Y}$ denote its closure. Let $\{U_j\}_{j\geq 1}$ be a collection of open sets in $\mathbb{R}^n$ such that $\bar{Y} \subseteq U_{j\geq 1} U_j$. Which of the following statements are true?

(a.) There exist finitely many positive integers $j_1, \ldots, j_N$ such that $Y \subseteq \cup_{k=1}^{N} U_{jk}$

(b.) There exists a positive integer N such that $Y \subseteq \cup_{j=1}^{N} U_j$

(c.) For every subsequence $j_1, j_2, \ldots$ we have $Y \subseteq \cup_{k=1}^{\infty} U_{jk}$

(d.) The exists a subsequence $j_1, j_2, \ldots$ such that $Y = \bigcup_{k=1}^{\infty} U_{jk}$

(77.) Let (a_n) and (b_n) be two sequences of real numbers and E and F be two subsets of $\mathbb{R}$. Let $E + F = \{a + b : a \in E, b \in F\}$. Assume that the right hand side is well defined in each of the following statements. Which of the following statements are true?

(a.) $\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$

(b.) $\limsup(E + F) \leq \limsup E + \limsup F$

(c.) $\liminf_{n\to\infty}(a_n + b_n) \leq \liminf_{n\to\infty} a_n + \liminf_{n\to\infty} b_n$

(d.) $\liminf(E + F) = \liminf E + \limsup F$

(78.) Let A be an $n \times n$ matrix. We say that A is diagonalizable if there exists a nonsingular matrix P such that PAP^{-1} is a diagonal matrix. Which of the following conditions imply that A is diagonalizable?

(a.) There exists integer k such that $A^k = I$

(b.) There exists integer k such that A^k is nilpotent

(c.) A^2 is diagonalizable

(d.) A has n linearly independent eigenvectors

(79.) Let f be an entire function such that $|zf(z) - 1 + e^z| \leq 1 + |z|$ for all $z \in \mathbb{C}$. Then

(a.) $f'(0) = -1$

(b.) $f'(0) = -1/2$

(c.) $f''(0) = -1/3$

(d.) $f''(0) = -1/4$

(80.) Let p be a prime number and N_p be the number of pairs of positive integers (x, y) such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{p^4}$.

Which among the following are possible values of N_p?

(a.) 0
(b.) 4
(c.) 5
(d.) 9

(81.) Let $\mathbb{T}$ denote the unit circle $\{z \in \mathbb{C} : |z| = 1\}$ in the complex plane and let $\mathbb{D}$ be the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$. Let R denote the set of points z_0 in $\mathbb{T}$ for which there exists a holomorphic function f in an open neighbourhood U_{z_0} of z_0 such that $f(z) = \sum_{n=0}^{\infty} z^{4n}$ in $U_{z_0} \cap \mathbb{D}$. Then R contains

(a.) All points of $\mathbb{T}$
(b.) Infinitely many points of $\mathbb{T}$
(c.) All points of $\mathbb{T}$ except a finite set
(d.) No points of $\mathbb{T}$

(82.) A positive integer n co-prime to 17, is called a primitive root modulo 17 if $n^k - 1$ is not divisible by 17 for all k with $1 \le k < 16$. Let a, b be distinct positive integers between 1 and 16. Which of the following statement are true?

(a.) 2 is a primitive root modulo 17
(b.) If a is a primitive root modulo 17, then a^2 is not necessarily a primitive Root modulo 17
(c.) If a, b are primitive roots modulo 17, then ab is primitive root modulo 17.
(d.) Product of primitive roots modulo 17 between 1 and 16 is congruent to 1 modulo 17.

(83.) Let $f = a_0 + a_1 X + \ldots a_n X^n$ be a polynomial with $a_i \in \mathbb{Z}$ for $0 \le i \le n$. Let p be a prime such that $p \mid a_i$ for all $1 < i \le n$ and p^2 does not divide a_n. Which of the following statements are true?

(a.) f is always irreducible
(b.) f is always reducible
(c.) f can sometimes be irreducible and can sometimes be reducible
(d.) f can have degree 1

(84.) Which of the following statements are true about subsets of $\mathbb{R}^2$ with the usual topology?

(a.) A is connected if and only if its closure $\bar{A}$ is connected
(b.) Intersection of two connected subsets is connected
(c.) Union of two compact subsets is compact
(d.) There are exactly two continuous function from $\mathbb{Q}^2$ to the set $\{(0,0),(1,1)\}$.

(85.) Consider the function $f(z) = \dfrac{(\sin z)^m}{(1-\cos z)^n}$ for $0 < |z| < 1$ where m, n are positive integers. Then $z = 0$ is

(a.) A removable singularity if $m \ge 2n$
(b.) A pole if $m < 2n$
(c.) A pole if $m \ge 2n$
(d.) An essential singularity for some values of m, n

(86.) Consider $A = \{1, 1/2, 1/3, \ldots, 1/n, \ldots \mid n \in \mathbb{N}\}$ and $B = A \cup \{0\}$. Both the sets are endowed with subspace topology from $\mathbb{R}$. Which of the following statements are true?

(a.) A is a closed subset of $\mathbb{R}$

(b.) B is a closed subset of $\mathbb{R}$

(c.) A is homeomorphic to $\mathbb{Z}$, where $\mathbb{Z}$ has subspace topology from $\mathbb{R}$

(d.) B is homeomorphic to $\mathbb{Z}$, where $\mathbb{Z}$ has subspace topology from $\mathbb{R}$

(87.) Let G be a group of order 24. Which of the following statements are necessarily true?

(a.) G has a normal subgroup of order 3

(b.) G is not a simple group

(c.) There exists an injective group homomorphism from G to S_8

(d.) G has a subgroup of index 4

(88.) For any complex valued function f let D_f denote the set on which the function f satisfies Cauchy-Riemann equations. Identity the function for which D_f is equal to $\mathbb{C}$.

(a.) $f(z) = \dfrac{z}{1+|z|}$

(b.) $f(z) = (\cos\alpha x - \sin\alpha y) + i(\sin\alpha x + \cos\alpha y)$, where $z = x + iy$

(c.) $f(z) = \begin{cases} e^{-\frac{1}{z^4}} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$

(d.) $f(z) = x^2 + iy^2$ where $z = x + iy$

(89.) Which of the following statements are true?

(a.) All finite field extension of $\mathbb{Q}$ are Galois

(b.) There exists a Galois extension of $\mathbb{Q}$ of degree 3.

(c.) All finite field extension of $\mathbb{F}_2$ are Galois

(d.) There exists a field extension of $\mathbb{Q}$ of degree 2 which is not Galois

(90.) For a positive integer n, let $\Omega(n)$ denote the number of prime factors of n, counted with multiplicity. For instance, $\Omega(3) = 1$, $\Omega(6) = \Omega(9) = 2$. Let $p > 3$ be a prime number and let $N = p(p+2)(p+4)$. Which of the following statements are true?

(a.) $\Omega(N) \geq 3$

(b.) There exist primes $p > 3$ such that $\Omega(N) = 3$

(c.) p can never be the smallest prime divisor of N

(d.) p can be the smallest prime divisor of N

(91.) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a nonzero smooth vector field satisfying $div\, f \neq 0$. Which of the following are necessarily true for the ODE $\dot{x} = f(x)$?

(a.) There are no equilibrium points

(b.) There are no periodic solutions

(c.) All the solutions are bounded

(d.) All the solutions are unbounded

(92.) The values of a,b,c,d,e for which the function

$$f(x)=\begin{cases} a(x-1)^2+b(x-2)^3 & -\infty<x\le 2 \\ c(x-1)^2+d & 2\le x\le 3 \\ (x-1)^2+e(x-3)^3 & 3\le x<\infty \end{cases}$$

is a cubic spline are

(a.) $a=c=1, d=0, b, e$ are arbitrary

(b.) $a=b=c=1, d=0, e$ is arbitrary

(c.) $a=b=c=d=1, e$ is arbitrary

(d.) $a=b=c=d=e=1$

(93.) Which of the following expression for $u=u(x,t)$ are solution of $u_t-e^{-t}u_x+u=0$ with $u(x,0)=x$?

(a.) $e^t\left(x+e^t-1\right)$

(b.) $e^{-t}\left(x-e^{-t}+1\right)$

(c.) $x-e^t+1$

(d.) xe^t

(94.) Consider the 2nd order ODE $\ddot{x}+p(t)\dot{x}+q(t)x=0$ and let x_1,x_2 be two solutions of this ODE is $[a,b]$. Which of the following statements are true for the Wronskian W of x_1,x_2 ?

(a.) $W\equiv 0$ in (a,b) implies that x_1,x_2 are independent

(b.) W can change sign in (a,b)

(c.) $W(t_0)=0$ for some $t_0\in(a,b)$ implies that $W\equiv 0$ in (a,b)

(d.) $W(t_0)=1$ for some $t_0\in(a,b)$ implies that $W\equiv 1$ in (a,b)

(95.) A mass m with velocity v approaches a stationary mass M along the x-axis. The masses bounce of each other elastically. Assume that the motion takes place in one dimension along the x-axis and v_f and V_f represent the final velocities of masses m and M along the x-axis. Which of the following are true?

(a.) $v_f=v, V_f=v$

(b.) $v_f=0, V_f=v$

(c.) $v_f=\dfrac{(m-M)v}{m+M}, V_f=\dfrac{2mv}{m+M}$

(d.) $v_f=\dfrac{mv}{m+M}, V_f=\dfrac{Mv}{m+M}$

(96.) Let $K(x,y)$ be a kernel in $[0,1]\times[0,1]$, defined as $K(x,y)=\sin(2\pi x)\sin(2\pi y)$. Consider the integral operator.

$\mathcal{K}(u)(x)=\int_0^1 u(y)K(x,y)\,dy$

where $u\in C([0,1])$. Which of the following assertions on $\mathcal{K}$ are true?

(a.) The null space of $\mathcal{K}$ is infinite dimensional

(b.) $\int_0^1 v(x)\mathcal{K}(u)(x)\,dx=\int_0^1 \mathcal{K}(v)(x)u(x)dx$ for all $u,v\in C([0,1])$

(c.) $\mathcal{K}$ has no negative eigenvalue

(d.) $\mathcal{K}$ is has an eigenvalues greater then $3/4$

(97.) Consider the Euler method for integration of the system of differential equations

$\dot{x}=-y$

$\dot{x}=x$

Assume that (x_i^n, y_i^n) are the points obtained for $i=0,1,\ldots,n^2$ using a time-step $h=1/n$ starting at the initial point $(x_0,y_0)=(1,0)$. Which of the following statements are true?

(a.) The points (x_i^n, y_i^n) lie on a circle of radius 1

(b.) $\lim\limits_{n\to\infty}(x_n^n, y_n^n)=(\cos(1),\sin(1))$

(c.) $\lim\limits_{n\to\infty}(x_2^n, y_2^n)=(1,0)$

(d.) $(x_i^n)^2+(y_i^n)^2>1$, for $i\geq 1$

(98.) Let B be the unit ball in $\mathbb{R}^3$ centered at origin. The Euler-Lagrange equation corresponding to the functional $I(u)=\int_B (1+|\nabla u|^2)^{1/2}\,dx$ is

(a.) $\operatorname{div}\left(\dfrac{\nabla u}{(1+|\nabla u|^2)^2}\right)^{1/2}=0$

(b.) $\dfrac{\nabla u}{(1+|\nabla u|^2)^{1/2}}=1$

(c.) $|\nabla u|=1$

(d.) $(1+|\nabla u|^2)\Delta u=\sum_{i,j=1}^{3} u_{x_i}u_{x_j}u_{x_i x_j}$

(99.) Let u be a positive eigenfunction with eigenvalue λ for the boundary value problem $\ddot{u}+2\dot{u}+a(t)u=\lambda u$, $\dot{u}(0)=0=\dot{u}(1)$ where $a:[0,1]\to(1,\infty)$. Which of the following statements are possibly true?

(a.) $\lambda>0$

(b.) $\lambda<0$

(c.) $\int_0^1 (\dot{u})^2 dt=2\int_0^1 u\dot{u}dt+\int_0^1 (a(t)-\lambda)u^2 dt$

(d.) $\lambda = 0$

(100.) Let $X = \{y \in C^1[0,\pi] : y(0) = 0 = y(\pi)\}$ and define $J : X \to \mathbb{R}$ by $J(y) = \int_0^\pi y^2(1 - y'^2)dx$. Which of the following statements are true?

(a.) $y \equiv 0$ is a local minimum for J with respect to the C^1 norm on X

(b.) $y \equiv 0$ is a local maximum for J with respect to the C^1 norm on X

(c.) $y \equiv 0$ is a local minimum for J with respect to the sup norm on X

(d.) $y \equiv 0$ is local maximum for J with respect to the sup norm on X

(101.) Let $u(x,y)$ solve the partial differential equation (PDE) $x^2 \dfrac{\partial^2 u}{\partial x \partial y} + 3y^2 u = 0$ with $u(x,0) = e^{1/x}$. Which of the following statements are true?

(a.) The PDE is not linear

(b.) $u(1,1) = e^2$

(c.) $u(1,1) = e^{-2}$

(d.) The method of separation of variables can be utilized to compute the solution $u(x,y)$

(102.) Consider the integral equation $\int_0^x (x-t)u(t)\,dt = x;\ x \ge 0$,

for continuous function u defined on $[0,\infty)$. The equation has

(a.) A unique bounded solution

(b.) No solution

(c.) A unique solution u such that $|u(x)| \le C(1+|x|)$ for some constant C

(d.) More than one solution u such that $|u(x)| \le C(1+|x|)$ for some constant C.

(103.) Let $\{(x_i, y_i) : i = 1,2,\ldots,n\}$ be given data points, where not all x_is are the same. C decides to fit a linear regression model of y on x with an intercept and D decides fit a linear regression model of y on x without an intercept. Let $\bar{x}$ and $\bar{y}$ be the sample means of x and y respectively. Which of the following statements regarding the two fitted models are NOT necessarily true?

(a.) Both the fitted regression line will pass through the point $(\bar{x}, \bar{y})$

(b.) C's fitted line will pass through $(\bar{x}, \bar{y})$, but D's fitted line will not pass through $(\bar{x}, \bar{y})$

(c.) for both the fitted models, the sample correlation coefficient between x_is and the corresponding residuals is zero

(d.) the sample correlation coefficient between x_is and the corresponding residuals is zero of C's fitted model, but not for D's fitted model.

(104.) Suppose $X_1,\ldots,X_n$ are i.i.d $N(\theta,1)$ where $\theta \ge 0$. Let $T = T(X_1,\ldots,X_n)$ be the maximum likelihood estimate of θ. Which of the following statements are true?

(a.) $E_\theta(T) - \theta \ge 0$ for all $\theta \ge 0$

(b.) $E_\theta(T)-\theta=0$ for all $\theta\geq 0$

(c.) $E_\theta(T)-\theta<0$ for all $\theta\geq 0$

(d.) There exists $\theta_0>0$ such that $E_0(T)-\theta<0$ for all $0<\theta<\theta_0$ and $E_\theta(T)-\theta>0$ for all $\theta\geq\theta_0$

(105.) Suppose X ~ Geometric $(1/2)$ (taking value in $\{1,2,3,...\}$) and conditional on X, the variable Y has Poisson (X) distribution. Similarly U ~ Poisson (1) and conditional on U, the variable V has Geometric $(1/(U+1))$ distribution. Then,

(a.) $E(Y)\geq E(V)$

(b.) $E(Y)\leq E(V)$

(c.) $Var[Y]\geq Var[V]$

(d.) $Var[Y]\leq Var[V]$

(106.) Consider an irreducible Markov chain with finite state space S. Let $p=((p_{ij}))$ be its transition probability matrix and let $p^n=((p_{ij}^{(n)}))$ denote n-step transition probability matrix for the chain.

Let $\alpha_{ij}=\lim_{n\to\infty}\frac{1}{n}\sum_{m=1}^{n}p_{ij}^{(m)}, i,j\in S$

Recall that the limit above always exists. Which of the following statements are necessarily true?

(a.) $\alpha_{ij}=\alpha_{kj}\ \forall i,j,k\in S$

(b.) $\sum_j\alpha_{ij}=1$ for all $i\in S$

(c.) $\alpha_{ij}>0$ for all $i,j\in S$

(d.) For all $i,j\in S$, the sequence $p_{ij}^{(n)}$ converges to α_{ij} as $n\to\infty$

(107.) Suppose $X_1,X_2,...,X_n$ are i.i.d. random variables which are uniformly distributed on the interval $(0,\theta)$ where $\theta>0$. Let $X_{(1)}\leq X_{(2)}\leq...\leq X_{(n)}$ be the corresponding order statistic. Consider testing the hypothesis $H_0:\theta=1$ versus $H_1:\theta>1$. Which of the following tests have significance level α for $0<\alpha<0.5$?

(a.) Reject H_0 when $X_1>1-\alpha$

(b.) Reject H_0 when $X_{(n)}>(1-\alpha)^{1/n}$

(c.) Reject H_0 when $X_{(n)}<(1-\alpha)^{1/n}$

(d.) Reject H_0 when $X_{(1)}>1-\alpha^{1/n}$

(108.) Consider a small clinical trial to study the effectiveness of a treatment for a particular illness, 10 patients are enrolled in this experiment. Let θ denote the probability that a randomly chosen partient in the population recovers from this illness due to this treatment. For a standard Bayesian analysis ,consider the Beta $(0.5,0.5)$ prior on θ (with density proportional to $(\theta(1-\theta))^{-1/2}$). Suppose exactly 6 out of the 10 patients recover. Which of the following are Bayes estimates of θ under the squared error loss function?

(a.) $\frac{13}{22}$

(b.) $\frac{11}{20}$

(c.) $\frac{1}{2}$

(d.) $E(\theta / 6$ out of 10 patients recovered)

(109.) If π is permutation of $\{1,2,\dots,n\}$, let $X_n(\pi)$ denote the number of fixed points , that is cardinality of the set $\{i \le n : \pi(i) = i\}$. If a permutation π is chosen uniformly at random then X_n is a random variable. Which of the following are correct?

(a.) $E(X_{25}) = 5E(X_5)$

(b.) $E(X_{25}) = E(X_5)/5$

(c.) $E(X_{25}) = E(X_5)$

(d.) $E(X_{25}) = [E(X_5)]^2$

(110.) Suppose we fit the linear model $Y = X\beta + \in$ using least squares where $Y = (Y_1, Y_2, \dots, Y_n)^T$ and $\in = (\in_1, \in_2, \dots, \in_n)^T$. Here X is a $n \times p$ non-stochastic matrix with full column rank and $\in_i$ s are i.i.d. with mean 0 and variance 1. For $i \in \{1,2,\dots,n\}$, let $\hat{Y}_i$ be the fitted value of Y_i and let $\hat{\in}_i$ be the corresponding residual. For $r, s \in \{1,2,\dots,n\}$, $r \ne s$ which of the following must be true?

(a.) The random variable $\in_r$ and $\hat{Y}_s$ are uncorrelated

(b.) The random variable $\in_s$ and $\hat{Y}_s$ are uncorrelated

(c.) The random variable $\hat{\in}_r$ and $\hat{Y}_s$ are uncorrelated

(d.) The random variable $\hat{\in}_s$ and $\hat{Y}_s$ are uncorrelated

(111.) Let $(X_1,Y_1),(X_2,Y_2),\dots,(X_n,Y_n)$ be i.i.d from a continuous bivariate distribution. Let R_i be rank of X_i among the X observation and S_i be rank of Y_i among the Y observation. Let Spearmen's statistic for testing independence between X and Y observation be denoted by T. Then, which of the following are true? Y

(a.) $T = \frac{12\sum_{i=1}^{n} R_i S_i}{n(n^2-1)} - \frac{3(n+1)}{n-1}$

(b.) $E(T) = 0$ when X and Y are independent

(c.) $Var(T) = \frac{1}{n-1}$ when X and Y are independent

(d.) $T \ge 0$

(112.) Let $(X_n, n \geq 1)$ and X be random variables defined on a common probability space, all having finite expectation and having characteristic functions $(\varphi_n, n \geq 1)$ and φ respectively. Which of the following

(a.) if $E(X_n) \to E(X)$ then there is at least one sample point w such that $X_n(w) \to X(w)$

(b.) if $X_n(w) \to X(w)$ for every sample point w then $E(X_n) \to E(X)$

(c.) if for every sample point w then $\varphi_n(t) \to \varphi(t)$ for all t

(d.) if $\varphi_n(t) \to \varphi(t)$ for all t then $X_n(w) \to X(w)$ for all at least one sample point w

(113.) Let $X_1, X_2, \ldots, X_n$ be random variable whose marginal distribution are $N(0,1)$. Suppose $W(X_i X_j) = 0$ for all $i, j, i \neq j$. Let $Y = X_1 + X_2 + \ldots + X_n$ and $V = X_1^2 + X_2^2 + \ldots + X_n^2$. Which of the following statements follow form the given condition?

(a.) Y has normal distribution with mean zero and variance n

(b.) V has chi-square distribution with n degree of freedom

(c.) $E(X_i^3 X_j^3) = 0$ for all, $i, j, i \neq j$

(d.) $P(|Y| > t) \leq \dfrac{n}{t^2}$ for all $t > 0$

(114.) Consider a Markov chain with state space $S = \{0, 1, 2, \ldots,\}$ and transition probabilities given as follows:

$p_{0,j} = 1/(j!e)$ for $j \geq 0$

$p_{i,i-1} = 1$ for $i > 0$ and i odd; $p_{i,i+1} = 1$ for $i > 0$ and i even. Which of the following are true ?

(a.) The chain is irreducible

(b.) The chain has period 2

(c.) There are infinitely many recurrent classes

(d.) zero is a transient state

(115.) A population of size N is divided into L strata of sizes $N_1, N_2, \ldots, N_L$ respectively. A strafified random sample of size n is drawn from the population where $n_1, n_2, \ldots, n_L$ denote the sample size in each of the L strata. Note that within each stratum units are chosen using simple random sampling .

Suppose the sample mean of the $j-th$ stratum is denoted by $\bar{y}_j$ and let $\bar{y}_{st} = \sum_{j=1}^{L} \dfrac{N_j \bar{y}_j}{N}$ and

$\sigma_{st}^2 = V(\bar{y}_{st})$

Consider a simple random sample of size n drawn from the population, independently of the first sample. Let $\bar{y}$ and σ^2 denote the sample mean and the variance of the sample mean for this sample. Which of the following statements are corrects?

(a.) $\bar{y}_{st}$ is an unbiased estimator of the population mean

(b.) $\bar{y}$ is an unbiased estimator of the population mean (c.) $\sigma_{st}^2 \le \sigma^2$

(d.) if $\frac{n_j}{N_i} = \frac{n}{N}$ for all, then $\sigma_{st}^2 \le \sigma^2$

(116.) Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be n independent observation from the uniform distribution on $S_\theta = \{(x,y) : \theta \le x^2 + y^2 \le \theta + 1\}$ where $\theta > 1$ is an unknown parameter. Which of the following statements are corrects?

(a.) $\max_{i \le i \le n}(x_i^2 + y_i^2) - 1$ is a maximum likelihood estimate of θ

(b.) $\min_{i \le i \le n}(x_i^2 + y_i^2)$ is maximum likelihood estimate of θ

(c.) Any value between $\max_{i \le i \le n}(x_i^2 + y_i^2) - 1$ and $\min_{i \le i \le n}(x_i^2 + y_i^2)$ is a maximum likelihood estimate of θ

(d.) Any value between $\max_{i \le i \le n}(x_i^2) + \max_{i \le i \le n}(y_i^2) - 1$ and $\min_{i \le i \le n}(x_i^2) + \min_{i \le i \le n}(y_i^2)$ is a maximum likelihood estimate of θ

(117.) Let $\{N_t, t \ge 0\}$ be a Poisson process with intensity parameter 10. Let T be an exponential random variable with mean 6, and independent of the Poisson process. Which of the following are true?

(a.) $P(N_T = k) = e^{-10T}(10T)^k / k!$

(b.) $E(N_T) = 60$

(c.) $(N_{T+t} - N_T; t \ge 0)$ is a Poisson process with intensity parameter 4

(d.) $(N_{T+t} - N_T; t \ge 0)$ is a Poisson process with intensity parameter 10

(118.) Consider the following Linear Programming Problem. Minimise $14x + 9y + 6z$ subject to

$2x + y + z \ge 8$

$3x + 2y + z \ge 10$

$x \ge 0, y \ge 0, z \ge 0$.

What is the optimal value of the objective function?

(a.) 52
(b.) 48
(c.) 50
(d.) 56

(119.) Consider a sequence of i.i.d. observations $X_1, X_2, \ldots$ from $N(0, \sigma^2)$. Which of the following are unbiased and consistent estimators of σ^2? i

(a.) $\sqrt{\frac{1}{2}\frac{1}{n}\sum_{i=1}^{n}|X_i|}$

(b.) $\sqrt{\frac{1}{n}\sum_{i=1}^{n}|X_i^2|}$

(c.) $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}$

(d.) $\sqrt{\frac{1}{2\pi}\left(X_1^2+X_\pi^2\right)}$

(120.) Let X be a non-negative random variable with $E[X]=1$. Which of the following quantities is necessarily greater than or equal to 1?

(a.) $E[X^4]$

(b.) $(E[\cos X])^2+(E[\sin X])^2$

(c.) $E[\sqrt{X}]$

(d.) $E[1/X]$